This conception of heat input could only be described by making use of instantaneously generated interior heat sources. Physically, this is accomplished by several means. One method is the instantaneous supplying of large amounts of electrical energy to a structure. This would allow each molecule of the structure to act as an interior heat source. A second and more important method of inducing vibrations caused by interior heat sources is the instantaneous dumping of radiation or radiation-type particles into a structure.

Consider, for example, the heat generated by gamma radiation that is applied instantaneously to an infinitely long cylindrical shell of thickness h.[6] The heat conduction in the plate is governed by the equation.[7]

$$(\partial^2T/\partial z^2) - (\partial T/k\partial t) = -W/K \tag{1}$$

where $T(z, t)$ is the temperature in the shell, k is the thermal diffusivity of the shell material, K is the coefficient of heat conduction for the shell material, z is the space variable across the shell thickness, t is the time variable, and $W(z, t)$ defines the distribution of interior heat sources in the shell material.

If the deposition of radiation into the structural material is assumed to be constant across the thickness and instantaneous in time,

$$W = W_0\, \delta(t) \tag{2}$$

where $\delta(t)$ is the Dirac function and W_0 is a known constant. Then the temperature in the structure will be

$$T = k\, W_0\, H(t)/K \tag{3}$$

where $H(t)$ is the unit step function. The surfaces of the shell are assumed to be insulated perfectly.

Further, consider the equation of motion governing the motion of an infinitely long shell in its purely radial mode due to a temperature input,[8]

$$\rho h(dw^2/dt^2) + (E_p/a^2)w = (E_p/a)(1 + \nu)\alpha_t\, T_0 \tag{4}$$

where

$$T_0 = \frac{1}{h}\int_{-h/2}^{h/2} T dz \qquad \text{and} \qquad E_p = \frac{Eh}{(1 - \nu^2)} \tag{5}$$

In Eq. (4), ρ is the density of the shell material, E is the modulus of elasticity, α_t is the coefficient of linear expansion, ν is Poisson's ratio, a is the radius of shell, and w is the deflection of the shell in the radial direction, which is taken as positive outward. Substitution of (3) into (4) yields a solution,

$$w(t) = a(1 + \nu)\alpha_t(k/K)W_0(1 - \cos\omega_0 t)$$
$$\omega_0{}^2 = \frac{E}{\rho(1 - \nu^2)a^2} \tag{6}$$

The stress in the shell under this condition of heat is

$$\sigma = [E/(1 - \nu)]\alpha_t(k/K)W_0 \cos\omega_0 t \tag{7}$$

It is evident that the use of instantaneously generated interior heat sources will be the practical method for inducing vibrations in structural members. This is based on the independence of the temperature (3) on any diffusion across the thickness.

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On the Existence of a Pressure Plateau in Pure Laminar Separated Flows

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THIS note calls attention to the existence of an important discrepancy among the published works which describe the characteristics of a pure laminar, shock-induced, adiabatic, separated flow. Gadd, Holder, and Regan[1] clearly indicated that the now well-known plateau pressure exists only when transition begins prior to flow reattachment. Thus, as shown in Fig. 1a, they defined as laminar the flow for which the pressure distribution exhibited a simple reflex; whereas, they noted (as typified in Fig. 1b) that a substantial region of constant pressure upstream of the ramp signifies that the flow is becoming turbulent during reattachment. Although Chapman, Kuehn, and Larson[2] substantiated quite generally Gadd's[1] conclusion regarding the critical importance of transition in governing the nature of the shock interaction, they did not make such distinctions for the pressure distribution upstream of the ramp. In fact, their results consistently suggest that the pressure plateau is characteristic of pure laminar separation and reattachment! Because of the lack of a general appreciation of this discrepancy in the literature, considerable confusion may occur regarding the influence of Reynolds number on the upstream extent of separation for flows initially laminar. For example, Lees and Reeves[3] compared theoretical pressure distributions for ramp-induced

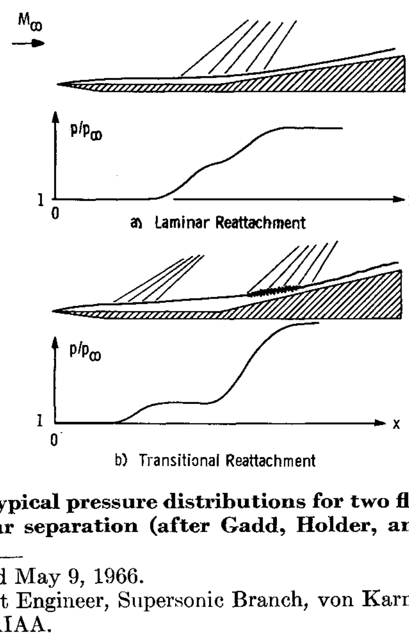

Fig. 1 Typical pressure distributions for two flow regimes of laminar separation (after Gadd, Holder, and Regan[1]).

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laminar separations with data from Fig. 30 of Chapman, et al.[2] They used these results to observe that ". . . as Reynolds number is lowered the separation point moves upstream. . . ." Similarly, Gray[4] was also led to classify as pure laminar all flow separations for which the upstream extent of interaction increased with decreasing Reynolds number. In both cases, the pressure distributions were typically like that in Fig. 1b, which, as previously noted, was classified by Gadd, et al. as transitional. Gadd, however, does not stand completely apart from this mainstream view, for one may infer from Kuehn's[5] work that the extent of a pure laminar separation decreases as the Reynolds number is decreased. This reversal in trend with Reynolds number, which is consistent with the implications of Gadd's[1] curves, is not directly supported, however, by any data or correlations such as those in Ref. 1. Although Kuehn's[5] definition of a laminar flow separation (and reattachment) is also consistent with Gadd's[1] pressure model (Fig. 1a), only in Ref. 1 was the pressure plateau specifically excluded. Therefore, only sketchy support for Gadd's flow model exists in the cited references.

But recently, however, some important test results in Ref. 6 became available. It appears that these are the only published pressure distribution data for laminar separations induced by ramps of substantially less than 10°. These data, which are in general agreement with Gadd's[1] model (Fig. 1a), also indicate that the upstream influence is at a maximum when the flow reattachment is laminar. Furthermore, it is noted in Ref. 7 that transition was triggered in the reattachment zone over a broad range of Reynolds number when the ramp angle was greater than approximately 10°. This later finding not only agrees with Gadd's[1] results, but it is generally consistent with the known influence of adverse pressure gradients upon laminar flow.

Consequently, since the data presented in Refs. 2–4 are for flow deflections (ramps, steps and flares) of 10° or more, it is indeed doubtful whether any of those data should be classified as laminar through reattachment. It seems evident, therefore, that an important question remains; can a pressure plateau ever exist when transition is downstream of flow reattachment?

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